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Supersymmetrization of radiation damping

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Abstract
We construct a supersymmetrized version of the model to the radiation damping introduced by the present authors (Mendes, Neves, Oliveira and Takakura 2005 *Preprint* hep-th/0503135). We discuss its symmetries and the corresponding conserved Noether charges. It is shown that this supersymmetric version provides a supersymmetric generalization of the Galilei algebra of the model. We have shown that the supersymmetric action can be split into dynamically independent external and internal sectors.

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1. Introduction

A fundamental property of all charged particles is that electromagnetic energy is radiated whenever they are accelerated. The recoil momentum of the photons emitted during this process is equivalent to a reaction force corresponding to the self-interaction of the particle with its own electromagnetic field, which originates radiation damping [1].

The process of radiation damping is important in many areas of electron accelerator operation [3], like in recent experiments with intense-laser relativistic-electron scattering at laser frequencies and field strengths where radiation reaction forces begin to become significant [4, 5].

In [2], the present authors presented a new approach in the study of radiation damping, introducing a Lagrangian formalism to the model in $D = 2 + 1$ dimensions given by

$$L = \frac{1}{2} m g_{ij} \dot{x}_i \dot{x}_j - \frac{\gamma}{2} \varepsilon_{ij} \dot{x}_i \ddot{x}_j, \qquad i, j = 1, 2, \tag{1}$$

where ε_{ij} is the Levi-Civita antisymmetric metric, g_{ij} is the pseudo-Euclidean metric given by

$$g = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2}$$

and where, as will be the case throughout the paper, the Einstein convention on the summation of repeated indices is employed. This formalism represents a new scenario in the study of this very interesting system. The Lagrangian (1) describes, in the hyperbolic plane, the dissipative system of a charge interacting with its own radiation, where the 2-system represents the reservoir or heat bath coupled to the 1-system [2]. The model (1) was shown to have the (2 + 1)-Galilean symmetry and the dynamical group structure associated with that system is the $SU(1,1)$ [2]. Note that this Lagrangian is similar to the one discussed in [6] (that is a special nonrelativistic limit of relativistic model of the particle with torsion investigated in [8]), but in this case we have a pseudo-Euclidean metric and the radiation-damping constant, γ, is the coupling constant of a Chern–Simons-like term. It is important to note that, despite the results obtained in this paper being very closely related with the ones from [7], the difference between them is not just the pseudo-Euclidean metric. The physical systems studied are different, where the constant γ is not a simple coupling constant, but depends on the physical properties of the charged particle, like its charge e and mass m, being related to the term in its equation of motion which describes an interaction of the charge with its own radiation field.

In this paper, we study a supersymmetrized version of the model (1), where we employ a supersymmetric enlargement of the Galilei algebra obtained in [2] and the supersymmetries of the model are determined. We also introduce the split into 'external' and 'internal' degrees of freedom of the supersymmetric model (1) in terms of new variables, where the radiation-damping constant introduces non-commutativity in the coordinate sector. The dynamic splits into the decoupled sum of the dynamics in the physical sector and in the auxiliary sector [2].

The paper is organized as follows. In section 2, we introduce the supersymmetric model and their canonical structure. In section 3, the Noether charges associated with the symmetries are obtained. In section 4, we have shown that the supersymmetric Lagrangian can be split into 'external' and 'internal' degrees of freedom and obtain the symmetries associated with each sector. In the final section, we present our concluding remarks and final comments.

2. The supersymmetric model and their canonical structure

To get the supersymmetric extension for this model, for the supersymmetric quantum mechanics $N = 1$, let us introduce a real field $X_i(t,\theta)$ with a Grassmann variable θ

$$x_i(t) \to X_i(t,\theta) = x_i(t) + \mathrm{i}\theta\psi(t). \tag{3}$$

Introducing the covariant derivative

$$D = \frac{\partial}{\partial\theta} - \mathrm{i}\theta\frac{\partial}{\partial t}, \tag{4}$$

we get the following supersymmetric extension from (1):

$$\begin{aligned}\bar{L} &= \mathrm{i}\int \mathrm{d}\theta\left(\frac{m}{2}g_{ij}\dot{X}_i DX_j - \frac{\gamma}{2}\varepsilon_{ij}\ddot{X}_i DX_j\right)\\ &= \frac{m}{2}g_{ij}\dot{x}_i\dot{x}_j - \frac{\gamma}{2}\dot{x}_i\ddot{x}_j + \mathrm{i}\frac{m}{2}g_{ij}\psi_i\dot{\psi}_j + \mathrm{i}\frac{\gamma}{2}\varepsilon_{ij}\dot{\psi}_i\dot{\psi}_j.\end{aligned} \tag{5}$$

Due to the presence of a second-order time derivatives in the Lagrangian, we have to introduce three momenta:

$$p_i = \frac{\partial\bar{L}}{\partial\dot{x}_i} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial\bar{L}}{\partial\ddot{x}_i} = mg_{ij}\dot{x}_j - \gamma\varepsilon_{ij}\ddot{x}_j, \tag{6a}$$

$$\tilde{p}_i = \frac{\partial\bar{L}}{\partial\ddot{x}_i} = \frac{\gamma}{2}\varepsilon_{ij}\dot{x}_j, \tag{6b}$$

$$\pi_i = \frac{\partial \bar{L}}{\partial \dot{\psi}_i} = -\mathrm{i}\frac{m}{2} g_{ij}\psi_j + \mathrm{i}\gamma\varepsilon_{ij}\dot{\psi}_j. \tag{6c}$$

This suggests that 12 canonical variables $\{x_i, \dot{x}_i, p_i, \tilde{p}_i; \psi_i, \pi_i\}$ should be employed. However, the elements in this set of canonical variables are not independent, because our model has two constraints (see equation (6b)) of the second class [9]

$$\Phi_i = \dot{x}_i + \frac{2}{\gamma}\varepsilon_{ij}\tilde{p}_j. \tag{7}$$

The Hamiltonian formalism for the Lagrangian (5) can be written in the ten-dimensional phase space $\{x_i; p_i; \tilde{p}_i; \psi_i; \pi_i\}$, using the Legendre transformation,

$$\begin{aligned} \bar{H} &= \dot{x}_i p_i + \ddot{x}_i \tilde{p}_i + \dot{\psi}_i \pi_i - \bar{L} \\ &= H_b + H_f, \end{aligned} \tag{8}$$

where the Hamiltonian for the bosonic sector, H_b (obtained in [2]), is

$$H_b = \frac{2m}{\gamma^2} g_{ij}\tilde{p}_i\tilde{p}_j - \frac{2}{\gamma} p_i \varepsilon_{ij}\tilde{p}_j, \tag{9}$$

and for the fermionic sector is

$$H_f = -\frac{\mathrm{i}}{2\gamma}\varepsilon_{ij}\left(\pi_i + \mathrm{i}\frac{m}{2} g_{il}\psi_l\right)\left(\pi_j + \mathrm{i}\frac{m}{2} g_{jk}\psi_k\right). \tag{10}$$

Next, we want to investigate the canonical equations of motion and the Poisson algebra of the model. But, due to the constraints (7) it is necessary to use the Dirac bracket [9]

$$\{A, B\}_D = \{A, B\} - \{A, \Phi_i\} C^{-1}_{ij} \{\Phi_j, B\}, \tag{11}$$

where A, B can be either bosonic- or fermionic-valued differentiable functions of the canonical variables $\{x_i, \dot{x}_i, p_i, \tilde{p}_i; \psi_i, \pi_i\}$ and the matrix C is defined through the relation $C_{ij} = \{\Phi_i, \Phi_j\}$.

In particular, the fundamental Poisson bracket relations are replaced by the symplectic structure depending on the choice of ten independent canonical variables. Choosing the independent variables as $y_a = \{x_i, p_i, \tilde{p}_i; \psi_i, \pi_i\}$, $a = 1, \ldots, 10$, we get

$$\{y_a, y_b\}_D = \omega_{ab}, \tag{12}$$

where

$$\omega = \begin{pmatrix} \mathbf{0} & \mathbf{1}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ -\mathbf{1}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \frac{\gamma}{2}\varepsilon & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{1}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{1}_2 & \mathbf{0} \end{pmatrix}, \tag{13}$$

with

$$\mathbf{1}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{14}$$

and $\mathbf{0}$ denotes the 2×2 null matrix.

The Hamiltonian equations of motion

$$\dot{y}_a = \{y_a, \bar{H}\}_D \tag{15}$$

take the form

$$\dot{x}_i = \{x_i, \bar{H}\}_D = -\frac{2}{\gamma}\varepsilon_{ij} p_j, \tag{16a}$$

$$\dot{p}_i = \{p_i, \bar{H}\}_D = 0, \tag{16b}$$

$$\dot{\tilde{p}}_i = \{\tilde{p}_i, \bar{H}\}_D = \frac{m}{\gamma} g_{ij}\tilde{p}_j - \frac{1}{2}p_i, \tag{16c}$$

$$\dot{\psi}_i = \{\psi_i, \bar{H}\}_D = \frac{\mathrm{i}}{\gamma}\varepsilon_{ij}\left(\pi_j + \mathrm{i}\frac{m}{2}g_{jl}\psi_l\right), \tag{16d}$$

$$\dot{\pi}_i = \{\pi_i, \bar{H}\}_D = -\frac{m}{2\gamma} g_{il}\varepsilon_{lj}\left(\pi_j + \mathrm{i}\frac{m}{2}g_{jk}\psi_k\right), \tag{16e}$$

where $\bar{H}$ is given by (8). To obtain the quantized form of the canonical commutation relation (15) as well as the Heisenberg equations of motion, we perform the replacement

$$\{y, y'\}_D \to \frac{1}{\mathrm{i}\hbar}[\hat{y}, \hat{y}'], \tag{17}$$

where $\hat{y}$, $\hat{y}'$ denote the quantized variables.

3. The Noether charges and their symmetries

Let us consider a Lagrangian $\bar{L}(x_i, \dot{x}_i, \ddot{x}_i; \psi_i, \dot{\psi}_i)$ which depends on the first and second time derivatives. The variation of the action $S = \int \mathrm{d}t\bar{L}$ under the change $x_i \to x_i + \delta x_i$ and $\psi_i \to \psi_i + \delta\psi_i$, bosonic and fermionic variables respectively, takes the form

$$\delta S = \int \delta\bar{L} = \int \mathrm{d}t\left(\delta x_i\frac{\partial\bar{L}}{\partial x_i} + \delta\dot{x}_i\frac{\partial\bar{L}}{\partial\dot{x}_i} + \delta\ddot{x}_i\frac{\partial\bar{L}}{\partial\ddot{x}_i} + \delta\psi_i\frac{\partial\bar{L}}{\partial\psi_i} + \delta\dot{\psi}_i\frac{\partial\bar{L}}{\partial\dot{\psi}_i}\right)$$
$$= \int \mathrm{d}t\frac{\mathrm{d}}{\mathrm{d}t}\left(\delta x_i p_i + \delta\dot{x}_i\tilde{p}_i + \delta\psi_i\pi_i\right), \tag{18}$$

from which we obtain the following formula for the generator:

$$C(t) = \delta x_i p_i + \delta\dot{x}_i\tilde{p}_i + \delta\psi_i\pi_i, \tag{19}$$

which is conserved $\left(\frac{\mathrm{d}}{\mathrm{d}t}C(t) = 0\right)$.

Let us list the generators of the symmetry for the Lagrangian (5):

(i) *Space translations*: $\delta x_i = \delta_i$, $\delta\dot{x}_i = 0$; $\delta\psi_i = \bar{\delta}_i$ ($\bar{\delta}_i$ are Grassmannian), $\delta\dot{\psi}_i = 0$, where δ_i and $\bar{\delta}_i$ are respectively the translation shifts of the bosonic and fermionic variables. So,

$$C_t = \delta_i p_i + \bar{\delta}_i\pi_i. \tag{20}$$

But the Lagrangian (5) is quasi-invariant under space-translation transformations; in fact,

$$\delta_t\bar{L} = \bar{\delta}_i\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{i}\frac{m}{2}g_{ij}\psi_j\right). \tag{21}$$

So the generators (20) are not conserved. However, as the nonconservation law takes the form

$$\frac{\mathrm{d}}{\mathrm{d}t}G(t) = \frac{\mathrm{d}}{\mathrm{d}t}\Lambda(t), \tag{22}$$

we can introduce modified generators $\tilde{G} = G - \Lambda$, which are conserved. In this case, we derive the following conserved generator:

$$\tilde{G}_t = P_i\delta_i + \Pi_i\bar{\delta}_i \tag{23}$$

where

$$P_i = p_i \qquad \text{and} \qquad \Pi_i = \pi_i - \mathrm{i}\frac{m}{2} g_{ij}\psi_j. \tag{24}$$

(ii) *Rotations*: $\delta_r x_i = -\varepsilon_{ij} x_j \phi_b$, $\delta_r \dot{x}_i = -\varepsilon_{ij}\dot{x}_j\phi_b$; $\delta_r\psi_i = -\varepsilon_{ij}\psi_j\phi_f$, where ϕ_b and ϕ_f are respectively the rotation angles of the bosonic and fermionic variables, then

$$G_r = -\varepsilon_{ij} p_i x_j \phi_b - \varepsilon_{ij} p_i \dot{x}_j \phi_b - \varepsilon_{ij}\pi_i\psi_j\phi_f. \tag{25}$$

Using the constraint equations (7), we find that

$$G_r = J_b\phi_b - J_f\phi_f \tag{26}$$

where

$$J_b = x_i\varepsilon_{ij} p_j - \frac{2}{\gamma}\tilde{p}_i^2, \qquad J_f = -\psi_i\varepsilon_{ij}\pi_j. \tag{27}$$

(iii) *Galilei boosts*: $\delta_{v_i} x_i = v_i t$, $\delta_{v_i}\dot{x}_i = v_i$; $\delta_{v_i}\psi = 0$ (note that the Grassmannian variables do not transform under Galilei boosts), then

$$G_{v_i} = v_i p_i t + v_i \tilde{p}_i. \tag{28}$$

But the Lagrangian (5) remains invariant up to a total time derivative under Galilei boosts transformation:

$$\delta_{v_i}\bar{L} = \frac{\mathrm{d}}{\mathrm{d}t}\left(m g_{ij} - \frac{\gamma}{2}\varepsilon_{ij}\dot{x}_i\right) v_i. \tag{29}$$

So using (22) we derive the following conserved generator:

$$\tilde{G}_{v_i} = \tilde{B}_i v_i \tag{30}$$

where

$$\tilde{B}_i = p_i t + \tilde{p}_i - m g_{ij} - \frac{\gamma}{2}\varepsilon_{ij}\dot{x}_i. \tag{31}$$

Using the constraint equations (7), we get the following conserved generator:

$$\tilde{B}_i = p_i t - m g_{ij} x_j + 2\tilde{p}_i. \tag{32}$$

(iv) *Time*: $\delta_\tau t = \tau$ (where τ is the translation shift of the time variables).

As the Lagrangian of the model (5) does not explicitly depend on time, the conserved quantity corresponding to the time translation is given by the Hamiltonian (8).

(v) *Supersymmetry*: $\delta_Q x_i = \mathrm{i}\epsilon\psi_i$, $\delta_Q\dot{x}_i = \mathrm{i}\epsilon\dot{\psi}_i$; $\delta_Q\psi_i = -\epsilon\dot{x}_i$ (where ϵ is an infinitesimal Grassmannian parameter), then

$$G_Q = \mathrm{i}\epsilon\psi_i p_i + \mathrm{i}\epsilon\dot{\psi}_i\tilde{p}_i - \epsilon\dot{x}_i\pi_i. \tag{33}$$

However, the Lagrangian (5) remains also invariant up to a total time derivative under supersymmetric transformation, so

$$\delta_Q\bar{L} = \epsilon\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{i}\frac{m}{2} g_{ij}\dot{x}_i\psi_j - \mathrm{i}\frac{\gamma}{2}\varepsilon_{ij}\dot{x}_i\dot{\psi}_j\right). \tag{34}$$

Using the relation (22) and introducing the constraint equations (7), we get the conserved generator

$$\tilde{G}_Q = Q\epsilon, \tag{35}$$

then

$$Q = \mathrm{i}p_i\psi_i - \frac{2}{\gamma}\varepsilon_{ij}\tilde{p}_i\left(\pi_j + \mathrm{i}\frac{m}{2} g_{jk}\psi_k\right), \tag{36}$$

where the Poisson algebra of this supercharge is given by

$$-\frac{\mathrm{i}}{2}\{Q, Q\}_D = H_b + H_f = \bar{H} \tag{37}$$

which is the Hamiltonian of the model (see (8)).

4. Supersymmetry in external and internal sectors

Now, we will show that the supersymmetric Lagrangian (5) can be split into 'external' and 'internal' degrees of freedom dynamically independent as in [2]. To this end, following Faddeev–Jackiw's method of describing Lagrangians with higher order derivatives [10], we describe, equivalently, the action (5) as

$$\bar{L}^{(0)} = \frac{1}{2}g_{ij}v_i v_j - \frac{\gamma}{2}\varepsilon_{ij}v_i\dot{v}_j + \frac{\mathrm{i}}{2}g_{ij}\psi_i\dot{\psi}_j + \mathrm{i}\gamma\varepsilon_{ij}\dot{\psi}_i\rho_j - \mathrm{i}\frac{\gamma}{2}\varepsilon_{ij}\rho_i\rho_j + p_i(\dot{x}_i - v_i), \tag{38}$$

where the field equation for ρ_i is purely algebraic and where ρ_i is fermionic (we put for simplicity $m = 1$).

Now, analysing the first-order Lagrangian $\bar{L}^{(0)}$, equation (38), from the symplectic point of view [10], the Dirac brackets among the phase space variables $\xi_a = \{x_i, p_i, v_i; \psi_i, \rho_i\}$ are

$$\{\xi_a, \xi_b\}_D = f_{ab}^{-1}, \tag{39}$$

where f_{ab}^{-1} is the inverse of the symplectic matrix

$$f = \begin{pmatrix} \mathbf{0} & -\mathbf{1}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{1}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \gamma\varepsilon & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathrm{i}g & \mathrm{i}\gamma\varepsilon \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathrm{i}\gamma\varepsilon & \mathbf{0} \end{pmatrix}. \tag{40}$$

So, for the fermionic sector of the Lagrangian (38), one obtains

$$\{\psi_i, \psi_j\}_D = 0; \qquad \{\psi_i, \rho_j\}_D = -\frac{\mathrm{i}}{\gamma}\varepsilon_{ij}; \qquad \{\rho_i, \rho_j\}_D = \frac{\mathrm{i}}{\gamma}g_{ij}. \tag{41}$$

The Dirac brackets for the bosonic sector are the same as the ones obtained in section 2.

In order to split (38) into external and internal sectors, the variables introduced in [7, 11] are modified as

$$\begin{aligned} &\mathcal{Q}_i = \gamma(g_{ij}v_j - p_i), && X_i = x_i + \varepsilon_{ij}\mathcal{Q}_j, \\ &P_i = p_i, && \tilde{\psi}_i = \psi_i - \gamma g_{ik}\varepsilon_{kj}\rho_j, \end{aligned} \tag{42}$$

giving the set of canonical Poisson brackets,

$$\begin{aligned} &\{X_i, X_j\}_D = -\gamma\varepsilon_{ij}, && \{P_i, P_j\}_D = 0, \\ &\{X_i, P_j\}_D = \delta_{ij}, && \{\mathcal{Q}_i, \mathcal{Q}_j\}_D = -\gamma\varepsilon_{ij}, \end{aligned} \tag{43}$$

for the bosonic sector. For the fermionic sector, the new fermionic variables satisfy the following Poisson brackets algebra:

$$\{\tilde{\psi}_i, \tilde{\psi}_j\}_D = \mathrm{i}g_{ij}, \qquad \{\tilde{\psi}_i, \rho_j\}_D = 0. \tag{44}$$

Now, substituting (42) into (38) the action takes the form

$$\bar{L}^{(0)} = \bar{L}^{(0)}_{\rm ext} + \bar{L}^{(0)}_{\rm int} \tag{45}$$

where

$$\bar{L}^{(0)}_{\rm ext} = P_i\dot{X}_i + \frac{\gamma}{2}\varepsilon_{ij}P_i\dot{P}_j - \frac{1}{2}g_{ij}P_iP_j + \frac{\rm i}{2}g_{ij}\tilde{\psi}_i\dot{\tilde{\psi}}_j, \tag{46}$$

$$\bar{L}^{(0)}_{\rm int} = \frac{1}{2\gamma}\varepsilon_{ij}\mathcal{Q}_i\dot{\mathcal{Q}}_j + \frac{1}{2\gamma^2}g_{ij}\mathcal{Q}_i\mathcal{Q}_j + {\rm i}\frac{\gamma^2}{2}g_{ij}\rho_i\dot{\rho}_j - {\rm i}\frac{\gamma}{2}\varepsilon_{ij}\rho_i\rho_j. \tag{47}$$

We see that our Lagrangian separates into two disconnected parts describing the 'external' and 'internal' degrees of freedom both describing supersymmetric models. Note that, while original coordinates commute, $\{x_i, x_j\} = 0$, both the 'external' and 'internal' positions, X_i and $\mathcal{Q}_i$, respectively, are non-commuting (see (43)).

Theses actions are invariant under the following set of supersymmetry transformations:

(i) for the external sector, equation (46), we get

$$\delta_Q P_i = 0, \qquad \delta_Q\psi_i = -\epsilon P_i, \qquad \delta_Q X_i = {\rm i}g_{ij}\epsilon\tilde{\psi}_j; \tag{48}$$

(ii) for the internal sector, equation (47), we get

$$\delta_Q\mathcal{Q}_i = {\rm i}\gamma\epsilon\varepsilon_{ij}\rho_j, \qquad \delta_Q\rho_i = \frac{1}{\gamma^2}\epsilon g_{ij}\mathcal{Q}_j, \tag{49}$$

where ϵ is a constant Grassmann number.

The supercharge corresponding to (46), generator for the transformations (48), is given by the formula

$$Q_{\rm ext} = J^0_{\rm ext} = \frac{\delta\bar{L}^{(0)}_{\rm ext}}{\delta\dot{\varsigma}_i}\frac{\delta\varsigma_i}{\delta\phi} = {\rm i}g_{ij}\tilde{\psi}_iP_j, \tag{50}$$

where $J^0_{\rm ext}$ is the conserved current and $\delta\phi$ is an infinitesimal parameter. The external Hamiltonian can be obtained consistently as

$$\bar{H}^{(0)}_{\rm ext} = -\frac{\rm i}{2}\{Q_{\rm ext}, Q_{\rm ext}\}_D = \frac{1}{2}g_{ij}P_iP_j. \tag{51}$$

Similarly, the supercharge corresponding to (47), generator of the transformations (49), is given by

$$Q_{\rm int} = J^0_{\rm int} = \frac{\delta\bar{L}^{(0)}_{\rm int}}{\delta\dot{\varsigma}_i}\frac{\delta\varsigma_i}{\delta\phi} = -\frac{\rm i}{2}\mathcal{Q}_i\rho_i, \tag{52}$$

and our internal Hamiltonian is given by

$$\bar{H}^{(0)}_{\rm int} = -\frac{\rm i}{2}\{Q_{\rm int}, Q_{\rm int}\}_D = -\frac{\rm i}{2\gamma^2}g_{ij}\mathcal{Q}_i\mathcal{Q}_j + {\rm i}\frac{\gamma}{2}\varepsilon_{ij}\rho_i\rho_j. \tag{53}$$

5. Concluding remarks

In this paper, we have presented a complete formulation of a supersymmetrized version of the model (1). We have started with the construction of the supersymmetric model (5) then, using the Dirac formalism for constrained Hamiltonian systems, the equations of motion and the canonical structure of the supersymmetric model are presented.

Next, we construct the (2 + 1)-dimensional Galilean supersymmetry and supersymmetry transformations of the variables appearing in the Lagrangian. Using Noether's procedure, we construct the conserved quantities associated with these symmetries, such as supercharges (36), which are the generators of supersymmetry transformations.

Finally, introducing non-commutative coordinates (42) and using Faddeev–Jackiw's method, we see therefore that the supersymmetric action (5) can be split into dynamically independent external and internal sectors (see (46), (47)). As we have shown, the external (46) and the internal (47) sectors remain invariant under the supersymmetric transformations (48) and (49), respectively, and the associated supercharge is constructed.

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